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Let us find out:
How they learn decimal numbers?

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Abstract :-

The purpose of this research project was to investigate the effects of a short-term teaching experiment on the learning of decimal numbers by primary students. The literature describes this area of mathematics as highly problematic for students.

The content first covered student understanding of decimal symbols, and how this impacted upon their ability to order decimal numbers and carry out additive operations. It was then extended to cover the density of number property, and the application of multiplicative operations to situations involving decimals. In doing so, three areas of cognitive conflict were encountered by students, the belief that longer decimal numbers are larger than shorter ones (irrespective of the actual digits), that multiplication always makes numbers bigger, and that division always makes numbers smaller.

The use of a microgenetic approach yielded data was able to be presented that provides details of the environment surrounding the moments where new learning was constructed. The characteristics of this environment include the use of physical artifacts and situational contexts involving measurement that precipitate student discussion and reflection.

The methodology allowed for the collection of evidence regarding the highly complex nature of the learning, with evidence of 'folding back' toofcompetingearl schema. The discussion presents reasons as to why the pedagogical approach that was employed facilitated learning.

One of the main findings was that the use of challenging problems situated in measurement contexts that involved direct student participation promoted the extension and/or re-organization of student schema with regard to decimal numbers.

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Introduction :-

Results of large scale surveys such as the National Assessment of Educational Progress (Carpenter et al 1981) in the United States of America and Concepts in Secondary Mathematics and Science (Brown 1981) in the United Kingdom show that children experience difficulty in learning and applying decimal number concepts. Investigations in New Zealand have revealed similar patterns (Carr 1986, Department of Education 1982). Research suggests that many children do not develop adequate meanings for decimal concepts because of a lack of conceptual knowledge, absence of connections between form and understanding and over-generalisation of fraction and whole number rules.

A child who has just been exposed to instruction on decimals must build a representation of decimal numbers and relate decimals to other number systems, notably whole numbers and fractions. Prior knowledge of whole numbers and fractions can both support and interfere with the construction of a concept of decimals. The over-generalizations or misconceptions are discussed under the following headings: Whole number knowledge and Fraction knowledge.

Problems that students have with the learning of decimal numbers are well documented in the literature. These include a lack of understanding of the meaning of the symbols used to represent decimal quantities (a semiotic problem), and a lack of facility in using them in mathematical operations.

Review of Literature :-

The central epistemological issues for the semiotic problem have been researched, with the findings available to the educational community for over twenty years (e.g. Sackur-Grisvard & Leonard, 1985). These findings describe how understanding of two referent systems (place-value and fractions) has to be combined in a reconstructive process in order to create meaning. Subsequent research has expanded upon (rather than conflicted with) this earlier work (e.g. Stacey & Steinle, 1999; Steinle & Stacey, 1998). For many years, the educational community also has had access to research that has documented the deep-seated difficulties learners of all ages have with situations involving operations with decimal numbers (e.g. Burns, 1990; Graeber & Tirosh, 1990). Despite the widespread knowledge about these difficulties, there is evidence to suggest that little has changed in terms of student achievement with decimals (e.g. Bana & Dolma, 2004, Young-Loveridge, 2007). The use of decimals is ubiquitous in the wider community and includes contexts such as financial and statistical literacy, measurement, and probability. Therefore, knowledge of decimal numbers is not optional for a society that desires all of its citizens to employ mathematics effectively in everyday life. Studies that have demonstrated effective pedagogical responses to the needs of students learning decimal place-value have fostered connections in student thinking between decimal symbols, concrete models and prior experiences (e.g. Helme & Stacey, 2000; Irwin, 2001).

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There are some studies that have investigated how students might be assisted to make sense of decimal numbers in situations that require the application of the four basic arithmetic operations. Examples include Bonotto (2005), and Irwin and Britt (2004). The fact that there are few studies that both present examples of successful interventions and provide models for explaining the process of the learning, has established a need to conduct more research in this area.

My involvement with the teaching of decimal numbers stems from a project I was involved with in 2011-2012. I sought to model to teachers the process of using diagnostic data to plan and deliver lesson sequences. Decimal numbers was a context I could use in classes from 5 to 9, and thus work with Primary and Secondary Schools. The interventional approached I used was influenced by reading recent research of the time, particularly Stacey, Helme, Archer and Condon (2001) and Irwin (2001). My experiences showed me that without attending to the reasoning behind the practice I was demonstrating, teachers were at best likely to copy what I had done, and at worst consider the approach as too different from 'proper'instruction. maths t

In 2012, I had the opportunity to research the area of decimal numbers in more depth as I engaged in a experiments at one school. This served as an end in itself, but also provided data that formed part of he present research.

Recent literature shows that educational researchers in mathematics have produced models that seek to explain the process of learning and to account for instances of nonlearning. Many of these models are situated in studies involving rational number (e.g.Simon, Tzur, Heinz & Kinzel, 2004). These studies do not focus on the examination of student misconceptions, but rather on understanding the processes that students are engaged in while learning, and the actions teachers can perform to enhance student cognitive development. In order to describe and examine these processes, student data is gathered while learning is taking place. This is in order to capture the conditions and conversations surrounding the moments when new connections were made and new thinking expressed. This positive attitude to students' priorhelp teachersknowledge improve their personal pedagogical content knowledge, resonated with my own beliefs about teaching. I wanted to contribute to the educational community in similar fashion. This desire shaped the nature of the data that I wanted to collect and thus determined the type of methodology that I went on to use. This research was a microgenetic study (Siegler, 2007) to examine how students' understandings of decimals can be enhanced by a short-term teaching experiment.

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Rational:-

The purpose of this research was to examine how models of learning are worked out with students in mathematics. The mental activities of building upon prior knowledge and the resolution of cognitive conflict are at the heart of a constructivist perspective of learning and form the basis of this study. Research in the last ten years has made us more aware of the complexity of the learning process, in particular as non-linear, recursive models are found to better fit observed student behavior than more simplistic models. There are calls from within the mathematics educational community to further examine the fine detail of learning in order to better understand the processes by which learning occurs. Studies that provide details of how students reconstruct their thinking are essential for the refinement of theories of learning. This refinement may result in more complete information being available for educators who may then alter their practice and so ultimately improve student achievement. This is especially pertinent in areas of mathematics that have proven problematic for students, as current practices are demonstrably not addressing key learning needs. This emphasis on *how* students are learning is most relevant where conceptual reorganization is identified as being the central issue, and not one of computational accuracy or factual recall.

In this research, the mathematical context within which these learning activities are explored is that of decimal numbers. An understanding of decimals is required for computational tasks within arithmetic and algebra, and is also necessary for work with measurement, geometry, and statistics (especially in probability). A growing awareness of the need for financial and statistical literacy also serves to underscore how important this area of mathematics is in terms of the numeracy level of the population as a whole. Poor attainment by students is well documented in the literature and is ascribed to a lack of conceptual understanding. This area of mathematics has been chosen because it is essential to student mathematical development and yet student success continues to be lowThe research community has responded to this need by work that has described the cognitive accommodations required of students and by reporting upon the partial constructions students have made as they have struggled to reorganize their thinking. However, while the conceptual issues surrounding the learning of decimals have been widely reported, there have been few studies that have documented both successful teaching innovations and described how these innovations have effected change in students' conceptual schema

Consideration of recent models of learning and the acknowledged need to improve the teaching of decimals form the niche within which this research is situated and provide its main purpose.

Research Question:-

How can students' understandings of decimals be enhanced by a short-term teaching experiment?

Participants:-

At the beginning of each phase of the study, the students were believed to be at about the same stage of mathematical development. The classroommake tea these decisions. In Phase 1, this meant that students were initially rated at Stage 5, and in Phase 2, at Stage 6 In brief, this implied that the Phase 1 students could add whole numbers by using at least one part-whole strategy, had started working with whole-number multiplication problems, and could order unit fractions. The Phase 2 students could add whole numbers using at least two strategies, knew most/all of their basic multiplication facts and could find a non-unit fraction of a set. The two groups of students were regarded as slightly above average in mathematics in their respective schools. Comparisons with national guidelines show them to be at - but not exceeding - expectations for their Year Levels.

Procedure: Design and Implementation :-

Design: Phase 1

It was clear from the literature that the ability of students to order decimal numbers was problematic. A specific objective in Curriculum (CBSE) was chosen, "Within a range of meaningful contextsithup studentsto3decimal).will placbea

That students would be required to *use* place-value knowledge of decimals is consistent with the view of knowledge described by Heinz et al (2011) and Hiebert et al (2006). Students would be asked to apply their knowledge to both contextual and non-contextual situations. A benchmark was needed that would provide a quantified measure of understanding. I had previously carried out extensive student trials (n > 200) of a diagnostic tool I had adapted from one produced by the University of Melbourne (Stacey & Steinle, 1999), the Decimal Comparison Test (DCT). Its use would provide this benchmark, with additional information coming from a task used in the NDP diagnostic interview and two tasks that ran parallel to those used in the literature describing common student misconceptions.

The instructional design provided for as little knowledge to be transmitted as possible. Instead, it aimed to get students to utilize their prior knowledge by connecting this with new contexts, and to generate new concepts via their interaction with challenging tasks. This choice of pedagogical approach was made in response to the literature that described student learning using constructivist frameworks and to models of learning consistent with these frameworks. Specifically, the intention was to engage students in using concrete materials to provide contexts within which to discuss the understanding and procedures required to order decimal numbers successfully. It was hoped that the use of decimal place-value could be extended to involve situations requiring the addition and subtraction of decimals and to the repeated addition model of multiplication. This potential learning trajectory is set out below.

Planned Learning Trajectory :-

Students will:

1. Understand fractions involving tenths.
2. Show they understand the iterative nature of fractions.
3. Recognise how tenths relate to the first decimal place and so start to interpret decimal symbols correctly.
4. Extend this understanding to include hundredths and so challenge the whole number view of decimals.
5. Generalize this concept to include any number of decimal places.
6. Begin working with decimal numbers using the operations of addition, subtraction, and multiplication (the latter only as repeated addition).

It was anticipated that students at Stage 5 would have working knowledge of these first two steps of this journey.

Implementation: Phase 1

This was held in August, 2011. Four, 45-minute periods of interaction were held in a medium-sized room away from the classroom. The final session was 30 minutes long. No lectures or notes were given to students. Instead, lessons began with tasks involving the use of manipulatives, with questions and conversations arising from these tasks. Games and written recording were employed to reinforce new learning. Questions being posed by either the teacher or the students provided the forward impetus into new areas of learning. The actual implementation of the planned learning journey was continually adjusted so as to accommodate responses to the formative evidence being collected.

The instructional pathway was as follows:

Day 1 Iteration of unit fractions with denominators up to tenths using materials, and then as a mental process.

Day 2 Introduction to the pipe numbers manipulative. Creating a situation where the paradox of whole-number thinking was exposed and resolved by the students.

Day 3 Exploration of decimal place-value using measurement tasks.

Day 4 Reinforcement of decimal numbers through the use of games. Extension of the concept of decimal place-value to addition.

Day 5 Using diagrams to represent decimal numbers. Further reinforcement using games.

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Design: Phase 2

Reading undertaken after the completion of Phase 1 convinced me that the problems students had with the use of decimals in additive contexts stemmed from place-value misconceptions, rather than being issues with the operation itself. For example, a student writing 3.4 + 3.21 = 6.25 is demonstrating understanding of the additive process, but misunderstanding the symbols used to describe the numbers. In response, I planned the next iteration to include work with additive contexts, but also to investigate how students would re-adjust their mental schema of multiplicative operations when faced with situations involving decimals. This re-conceptualizing was described as highly problematic in the literature.

I had become more convinced of the efficacy of using physical materials in measurement contexts as a result of the literature survey and my experiences in Phase 1. Examination of the use of these artifacts to promote learning in the context of multiplicative work with decimals became the investigative focus. The subtle shift in thinking involved is expressed in the following statements. In Phase 1, I wanted to see if the use of materials and realistic contexts could result in cognitive reconstruction, whereas in Phase 2, I wanted to investigate how the use of materials and contexts affected the reconstructive process.

It was intended that students would take part in a variety of experiences that would serve to expand their existing knowledge of decimal numbers. This was formalized as below.

Planned Learning Journey :-

Students will:

1. Demonstrate that they can use their understanding of decimal place-value to order decimal numbers and then to add and subtract them.
2. Show that they have generalised the decimalization process through situations involving the 'density of number' property.
3. Be able to solve single-digit multiplication and division problems involving simple decimal numbers.
4. Begin to extend their multiplicative knowledge to more difficult problems involving decimals including double-digit multiplication and finding fractions of fractions.

Implementation: Phase 2

This was held in December, 2011. Eight periods of interaction occurred. Various in-school factors meant that the learning sessions were held in three different rooms, at three different starting times and for periods of time ranging from 25 minutes to 1 ¼ hours. Flexibility was essential. For example, the water-pouring session had to be held on Day 3 as a suitable room was available then.

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Again, no formal teaching took place, but a continued emphasis on engagement with practical tasks and discussion filled most of the learning sessions. A slightly higher proportion of time than in Phase 1 was spent on completing written tasks and in student recording of their practical results.

Another contrast with Phase 1 was that the effect of mood on student engagement was noticeably more pronounced, perhaps reflecting the developmental age of the students. The existence of off-task discussions regarding peers, music, television, and school would not surprise teachers of this age group. In particular, Wani was 'growledDPfor at'uniform infringementsbytheon two occasions while walking to learning sessions and was disengaged for the beginning of each of those corresponding periods. At other times the group was totally focused and seemed very aware of the learning they were engaged in over and above the tasks they were performing.

Instructional Pathway:-

Day 1 Collection of initial data, introduction to the pipe numbers, decimal addition using this model.

Day 2 Decimal addition and subtraction using linear and area models.

Day 3 Division using quantities of water and area as models.

Day 4 (Short session). Review of earlier tasks. Density of numbers via biscuit tasks. Multiplication of a decimal number.

Day 5 (Relatively short afternoon session). Density via number-line tasks and biscuits. Review of previous contexts.

Day 6 Quotitive division using string lengths. Density using number-lines.

Day 7 Further work with string contexts. Multiplication by a decimal number. Estimation of products using rounding.

Day 8 Double-digit multiplication with whole numbers and then decimals using an area model.

Data Collection :-

Different types of data were gathered for this research. Diagnostic data was collected in order to shape the intervention, as this information would allow student prior knowledge to be built upon.

In Phase 1, diagnostic data was captured through the Decimal Comparison Test (DCT), and the addition tasks as mentioned earlier. A group interview was held prior to the intervention phase. The children had previously agreed to have individual discussions but become apprehensive when the audio recorder was produced. They agreed to a compromise whereby the group discussion was recorded.

In Phase 2, initial data Was collected by the completion of written tasks. These tasks included a few diagnostic items and others whose purpose was to provide exposure to a range of multiplicative contexts involving decimals.

Samples of student work were collected during both phases of research. During Phase 1, this was achieved by the collection of student work done on paper. In Phase 2, an exercise book was provided for each student in order that all drawings and calculations could be recorded and easily collected. At the end of each teaching session, I made brief field notes on what had occurred. These included informative details (such as the timing and rooms in Phase 2), and any important non-verbal data(such as notes on student confidence). This writing also served to activities in order to prepare for the next session

.**Measure of Change Data:-**

At the end of Phase 1, evidence of the changes students had made in their thinking was obtained by the completion of written tasks and short, audio-taped interviews. Students were asked to complete a new DCT and to review their answers to the previous written tasks. They were also asked to complete two new tasks that required them to consider the ordering of decimal numbers in new contexts. Unlike the initial interview, each of the final interviews was on an individual basis. This no longer seemed to be an issue to the students. In these interviews, students were asked to compare their initial and final DCT papers (both of which were unmarked), and comment upon the reasons why changes had been made.

At the end of Phase 2, only written evidence could be collected post-intervention.

Comparison of Students' Initial and Final Responses to DCT

	Initial Result		**Final Result**	
NAME	**Code**	**Exception Count**	**Code**	**Exception Count**
Mani	No pattern	n/a	Money level	3
Grace	Longer is larger	0	Task Expert	1
Wani	Shorter is larger	3	Task Expert	0
Arahan	Shorter is larger	4	Task Expert	0
Rashmi	Longer is larger	0	Task Expert	2
Tarun	Longer is larger	0	Task Expert	1

Students' ResponsesNumberLineTask to

	Mani	**Grace**	**Wani**	**Arahan**	**Rashmi**	**Tarun**
Correct answer 8.6 or 8 6/10	6	No answer	8.6	14	No answer	8.6

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Students' Responses to the Task Ordering of Decimal Numbers Task

Task	Mani	Grace	Wani	Arahan	Rashmi	Tarun
Which is the larger of these two numbers? **2.6 2.27**	2.6	2.6	2.6	2.6	2.6	2.6
How do you know which is the larger?	0.6 is bigger than 0.2	Not sure	0.6 = -6 0.27 = -27	2.27 is larger (Contradicting herself)	Not sure	2.6 is like 260, 2.27 is like 227

Students' Responses Density of Number Task

Task	Mani	Grace	Wani	Arahan	Rashmi	Tarun
Show numbers between 1.6 and 1.7	Correct examples given	Correct examples given	Correct examples given	Correct examples given	Correct examples given	Correct examples given
7.578 and 7.579	Incorrect example	Correct examples	Incorrect example	Correct examples	Correct examples	Correct examples

Comparison of Students'Initial and Final Responses to Fractional Additional Task

	3/10 + 4/10		1.3 + 1.13	
	Initial	**Final**	**Initial**	**Final**
Mani	27	7/10	Left blank	1.33
Grace	7	7/10	2.16	2.43
Wani	Left blank	7/10	2.16	2.43
Arahan	Left blank	7/10	2.16	2.43
Rashmi	Left blank	Left blank	Left blank	2.16
Tarun	Left blank	7/10	2.16	2.16

Comparison of Students' Initial and Final Responses to Decimal Task

Task	Timing	Mani	Grace	Wani	Arahan	Rashmi	Tarun
4.2 x 2.6	Initial	6.8	9.2 4 x 2 = 8 2 x 6 = 12	9.2	6.8	9	Left blank
Answer 10.92	Final	6.8	8.8	8.4 Area Model	9.2	10.92 Area Model	10.92 Area Model

Findings:-

The research resulted in three main findings, with important implications for teachers and suggested directions for future research.

1. The learning sessions were designed around pedagogical actions where the learning of decimals relied upon measurement contexts. This provided students with experiences where meaningful enactments of tasks, combined with the cognitive challenge of new, student-generated data, promoted the re-shaping of their thinking. Students were afforded a degree of agency in their operation within these tasks, which allowed them to raise questions and reflect on the results of their actions at their own pace. This approach is consistent with recommendations made by earlier research and also helps address longstanding pedagogical issues regarding the teaching of decimal numbers. The research literature has established that student achievement in many applications of rational number is still of Major concern. An innovative approach that can demonstrate mechanisms whereby students address many of the problematic issues surrounding decimals is therefore worthy of careful consideration.

2. Having high expectations of the students coupled with the teaching mechanisms described above enabled significant learning to occur in relatively short time-frames. Generating learning involved the engineering of situations that were engaging both in terms of interest for students and with regard to important mathematical ideas. Scaffolding was initially required in order for students to complete tasks and its removal was important to gauge independent student knowledge. Students appeared to enjoy both the experiences of being challenged as well as their resulting successes. In the learning sessions, the practice of making explicit links between prior knowledge/experience and new learning was considered to be more important in facilitating student learning than close attention to student levels of learning. The careful consideration of pedagogical options was to

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result in activities that overcame many of the and students' compensated for their cognitive being a stage below confl where this content would normally be taught. The students invented new strategies and thought their way through challenging concepts in response to the high demand of the tasks before them in the context of a supportive environment.

3. Close observation of student learning was achieved by the use of a microgenetic study. This methodology provided instances of the complexity of student learning as described by recent theoretical models. Evidence was obtained of temporary and simultaneously competing constructs, recursion, and movement towards abstraction. Data from oral, enacted, and written sources would at times complement and at times contradict each other. Student learning in this research was idiosyncratic and non-linear, and generalization appeared to be an evolving process. The methodology allowed for this complexity to be captured so that it might be examined and reflected upon. It also accommodated changes made to planned teaching sequences in order to be contextually responsive to the students'- andimmediatepost-interventionmeasuresquestionsof success served to set the in-situ learning data into its educational context.

Implications:-

Two of the innovations used in the research have not been reported in the literature previously. The purposeful use of the pipe numbers equipment in measurement tasks goes beyond the reports of the use of this, and similar tools. In previous accounts, the purpose was to provide static representations of decimal numbers. Here, the actions of measuring actual objects provided the opportunity for a range of decimal issues to be discussed as they were perceived to be important by the students as well as providing student-generated sets of data that could be used to reinforce the consideration of place value. Prior to this study, the action of engaging students through the pouring of water in order to enact quotitive division involving decimal numbers has not been documented. These activities resulted in high levels of engagement by the students and had both epistemic fidelity and transparency. They appeared to promote learning, and thus can be described as counterexamples to expansive generalizations that are both pivotal and bridging. These innovations could be part of a scaled-up study to determine their robustness, as they may prove to have similar efficacy in other settings.

Of wider interest is the overall approach of using measurement contexts to engage students both in completing tasks and considering the mathematical ideas implicit in those tasks. When the use of manipulative materials has been criticized, it is often because the materials have been seen by teachers and/or students as ends in themselves. Tasks involving measurement situations provide motivation for students in that there is a self-determined agenda being enacted, i.e. the students want to find something out (a measure) and in doing so encounter new learning issues.

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Measurement also provides meaningful interaction with the continuous model of number in which decimals (and other fractions) are naturally found. This is an alte and to the Curriculum division' with of little mathematics overla strands.

There is an implicit challenge provided to the current practice of teaching students according to their 'stage'. This study provides a record of how th showed success at higher levels, but also relish tasks. These results support my conviction that delaying exposure to problematical ideas in mathematics is not always in the best interests of students. Instead, more attention could be given to the professional development of teachers to give them the confidence and ability to address these mathematical issues with their students. Making teachers more aware of examples of successful interventions and the models of learning that explain these successes may improve teacher pedagogical content knowledge and ultimately raise student achievement. While teachers may be generally aware of the need for having high expectations of their students, they may not be acquainted with many instances of lesson sequences that model this in mathematics. This research serves as an example of how the principle of having high expectations may be enacted in work with students.

While the intent of the research was to investigate the learning process, the study also provides a record of the teaching process of providing students with challenging tasks and appropriate concrete materials and how this approach generated learning. Many teachers have not had the experience of observing a sequence of lessons presented with this pedagogical approach. While the Curriculum provides opportunity for facilitator modeling, this is nearly always on a single-lesson basis. Many published research articles document the results of such pedagogical practice, and journals aimed directly at teachers tend to describe single innovative lessons. Having these sequences of lessons to examine and critique provides an informative source for teachers who want to learn more about the learning process, particularly as it applies to this problematic area. The revised curriculum requires teachers to investigate their own practice, and ways changes to their practice have an impact on student learning.

Directions:-

The routes to numeracy (other than through number) require further study of the literature and further investigation with students. This is not a case of rejecting the goal for numeracy, but a suggestion that the mechanism of achieving this (number-first) may be too simplistic. In particular, the interaction between continuous models of number and measurement needs further exploration. Each facet of the multiplicative work warrants its own study. This research was unable to investigate specific contexts deeply, but provided examples of potential lines of inquiry to address subtle inherent epistemological distinctions such as the difference for students in addressing situations of whole number × decimal versus decimal × whole number.

This study serves as an example of the microgenetic approach. The type of data produced offers both insights into learning that other methodologies cannot provide and a degree of connectedness with teacher experience that may prove to be engaging - and therefore informative - for this community. The evidence of in-situ learning produced in this study may be combined with the results from other microgenetic studies from different contexts and different aspects of mathematics, allowing meta-analysis to take place. This may result in general themes emerging that could shape future learning models and future teacher practices.

Further studies exploring how models of teaching that feature high expectations coupled with a practical problem-solving approach, might impact upon student achievement have the potential to provide information upon which recommendations regarding best practice can be formed.

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